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Tarski and Modality

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Abstract: Subscription to meta-language for self-referencing statements has been put forth by Tarski as solution to the Liar Paradox, suggestively indicative of the existence of higher order system through logic and language capable of substantiating truth. The paper aims to argue that the same methodology can be used to attribute truth values to propositions otherwise subsumed by many-valued logic. In many-valued logic, answers yield probabilities and multivalues versus definite truth values found in classical logic, yet this proves to be paradoxical as probabilities as a whole can be deemed to be either true or false. Wave function collapse yield stochastic states of affairs based on plugged eigenvalues, eliminating true from false states of affairs. Due to its aleatory nature, establishing truth values comes after decoherence, ensuing to probabilistic arguments such as the infamous Schrodinger's cat-in-a-box experiment's results, which is evaluated in terms of probability. Classical truth values "true" and "false" can thus be retained provided that the principles of non-contradiction and excluded middle are relaxed. To argue that a state of affair (post-decoherence state) is true is to coordinate spacetime dimension along with indicative markers of possible world actualized via modalities; to state however that something is true in all possible world does not necessitate subscribing to quantum logic. Truth thus transcends multivalued truth as meta-language trumps object language, as modalities encompass quantum logic.

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INTRODUCTION

There is nothing more daunting a task than decoding a notion so immediate yet so inconclusive as the truth. In the hopes of eliminating unnecessary baggage several philosophers decided to reduce the concept into merely an elaboration of a definite state of affairs, so that to say that "'Snow is white' is true" is nothing but a long hand version of the statement "Snow is white". Cognitively speaking thus, to say that a statement 'is true' adds nothing to the meaning and comprehension of the statement.[1] The utterance, by extension, of statement "'Snow is white' is true" is true' in effect poses redundancy. Such statement is merely an empirical, observable claim about actual state of affairs, and is susceptible to perpetual referencing. The idea is more commonly known as the Deflationary Theory of Truth[2] and is endorsed most notably by Quine (1970), dismissing truth as eliminable and unworthy of being a predicate.

As tempting as it is to dissolve the notion of truth as simply a linguistic vestige, this does not in any way capture the myriads of implications appended to the term. A minimalist theory of truth does not bode well if we intend to uphold the Aristotelian sentiment of bridging reality and truth[3], especially with reality treading far more obscure paths in the recent times. Alfred Tarski (1944) offered a method by which the truth of a statement is not merely contained in its own system but instead

[1] This however is notably different when the statement "Snow is white is false" is uttered, as falsity, as with apopathism, offers a different perspective that needs articulation, as opposed to its binary counterpart.

[2] Richard Schantz identifies at least four strands of the theory in his 1998 paper "Was Tarski a Deflationist?" For the purpose of our discussion, these theories would be lumped together under the umbrella term "Deflationary Theory of Truth" and would be treated interchangeably.

[3] Aristotle, and W. D. Ross, *Aristotle's Metaphysics*. (Oxford, England: Clarendon Press, 1981). *Metaphysics* 1.7,1011b25

ascribes it to a metalanguage in order to ascertain truth value of otherwise antinomous statements found within said system. This would imply that a statement's being 'true' ascribes its truthfulness to a higher language. Contrary to an apparent claim by the Disquotationalist School of Deflationary Theory, Tarski is clearly not a deflationist[4]—it is, at its best, a higher order correspondence theory of truth bent on hinging truth with satisfaction.

The birth of quantum mechanics stemming from the early 20th century confirmed one of the most misattributed quotes from Aristotle.[5] Reality has begun to distance itself away from two of the three main laws in logic; principles of excluded middle and contradiction are put to a test; and the cat, as is implied by Schrodinger, is both dead and alive. In the advent of multi-valued and probabilistic logic, the dichotomy of truth and falsity are expanded in various measures ranging from finite to infinite values. How then would we make sense of such truth? And would Tarski's initial methodology hold true despite these developments?

This paper argues of Tarski's metalanguage's efficacy in making sense of multi-valued statements. It is thus not senseless nor meaningless to say that '"*p*' is both true and false' is true', for it invokes a higher order metalanguage which encapsulates statement *p*'s nature of being both true and false at the same time. Albeit proportionally reduced in terms of probabilistic estimates, truth in metalanguage corresponds (i.e., bears the same weight) as the truth as perceived in the classical sense. Should there be any event that necessitates a probabilistic truth in metalanguage, a second order metalanguage thus would assume the previous domain, and so forth.

[4] Richard Schantz, "Was Tarski a Deflationist?," *Logic and Logical* Philosophy, 6 (1998), 157-172.

[5] "Let this then suffice to show that (1) that the firmest beliefs and that the opposite assertions are not true at the same time, (2) what happens to those who speak this way, and (3) why people do speak this way" pertaining to the Principle of Non-Contradiction. See *Metaphysics* 4.6,1011b13–15.

The first section provides a rundown on truth in language and refers to Kripkean notion of rigid designators in pertaining to possible world alternatives, while the second part is a short primer on quantum mechanics and its offshoot logic. The third section discusses truth in terms of modal worlds, while the last part attempts to weave the notions together to show that, indeed, quantum and probabilistic logic are subsumed by Tarskian notion of metalanguage.

TRUTH IN LANGUAGE

"To say of what is that is not, or of what is not that is, is false, while to say of what is that is, or of what is not that is not, is true."

So goes Aristotle's famous conclusion on the notion of truth in the Metaphysics. Clearly, the ontological quiddity pertained by above statement speaks of the state of affairs in the world. Originally, truth functions as a linguistic armament to align itself with reality, and while the definition took an unexpected turn in the last couple of centuries, the *telos* of truth remains the same. Two major theories contend for superiority—coherence theory and correspondence theories—although for the sake of discussion we shall focus on correspondence theory and give proper nod to coherence theory whenever necessary. Determining the better theory is left out for future discussion and is not to be scoped by this paper.

Recapitulating, only declarative statements are eligible of being attributed truth values. Russellian and Fregean notions attribute truth to a statement should the definiens tally with its *definiendum* and vice versa, and if, upon hinging resulting statement in reality, corresponds to it. Several paradoxes were raised in the process,[6] distinguishing the two concepts from each other and, in Russell's case, acknowledging the limitations of language and logic. One of the most notable points raised in their attempt to be succinct is the preciseness of language, disregarding

[6] See Bertrand Russell, "On Denoting," *Mind,* 14:56 (2005), 873 - 887.

vague and ambiguous statements should they not be able to correctly identify their subject in the corporeal world. This later arise several objections as to the truth of commonplace statements. For instance,

1 It will rain tomorrow.

Although declarative in nature, is compromised due to the ambiguity of the term "tomorrow". The term is indicative of a futuristic sentiment; as the event cannot be confirmed nor denied at the exact moment of utterance due to unrealized reference, it holds no real-life counterpart and thus, is not eligible for truth values. In sum, statement "It will rain tomorrow" has no truth value; "It will rain tomorrow because it's been raining for three days now" is obviously wrong, although we cannot say anything about its validity (we can only surmise that the reasoning is wrong and, since this is an example of inductive method, *incogent* to say the least). There is no deductive, non-artificial, and causal instance where the conclusion is held or can be held as true.

In lieu of a futuristic statement such as (1), one can effectively shift to a probabilistic statement, in which case the treatment of above engenders several truth values in the form of probabilities. This will be discussed in greater detail in the ensuing sections.

It is to be noted that Tarski's proposition came in light of the self-referencing dilemma. The statement,

2 This statement is false.

violates the law of non-contradiction altogether; while truth can be said of the statement's being false (i.e., that is, it is true that it is false), its context directly points to its being false. Thus, the paradox of the universal truth machine holds. Tarski was able to resolve the dilemma through a proposition of a metalanguage which scopes all possible statements/ combinatory features of the object language. Aside from its justificatory

and warranting power over statements similar to above, not much is known about metalanguages. However, it is asserted that a metalanguage should at the very least be richer than the object language so that every word in the object language has a corresponding (higher-ordered) word in the former. Ensuing justifications are to be tackled in the subsequent sections.

In the advent of possible worlds, probabilistic statements take center stage as physicists and philosophers alike gauge what can be said of such given our current outtake on the world. As is peculiar, probabilistic claims feature strands of multi-valued truth function. While plottable against the truth-falsity spectrum, the remaining area within denotes that there is an infinity of values between said two polar points. This event is generally acknowledged to occur in our actuality, rendering percentages of truth. On the other hand, utterances similar to Hilary Putnam's twin earth experiment[7] are not merely probabilistic in nature and belong to a different realm of possible worlds. Rigid designators, a proposed solution to above conundrum, draws its truth value from meaning and linguistic use rather than causality.

A PRIMER ON QUANTUM MECHANICS

Alluding to the complex intricacies of quantum mechanical probabilistic statements, famed physicists Richard Feynman was quoted saying that nobody understands quantum mechanics. Suffice to say that establishing truth in a system whose axioms are not fully determined, and experiences mar the observed response does not bode well with the classic coherence and correspondence theories of truth. What then constitutes to one's being

[7] Putnam's twin earth thought experiment illustrates a linguistic theorem wherein different speakers might subscribe to the same definitions, beliefs, and mental ideas or images regarding a particular object but still attribute different meaning toward said (external) object. This is manifested in his defense of the semantic externalist approach to meaning, which greatly coheres with a modal realist view of possible worlds. See Putnam, Hilary. "Meaning and Reference." *The Journal of Philosophy*, vol. 70, no. 19, Journal of Philosophy, Inc., 1973, pp. 699–711, https://doi.org/10.2307/2025079.

true? In response there has been a predilection in preferring probabilistic statements over single truth-functional statements. In cases where the probability is not equal among options, it is thus rational to assume that the most likely outcome would occur based on the higher percentage. However, certain complications occur in predicting should more than two options exist. For instance, if three options A, B, and C are all likely to occur at 33% each, it also follows that each choice is 67% unlikely to take place in reality. For the false dimension, one might adopt Tarski's treatment of a self-referencing false statement, modifying it to indicate the percentage of the statement's likelihood to be false. We leave the discussion of statistical interpretation to the statisticians; however, we are interested in the ensuing statement above condition would produce.

Regardless of their likelihood, probabilistic statements can be reduced into a mere question of truth and falsity if the same method used by Tarski is employed. Taking for instance Schrodinger's cat, one of the original statements is:

3 The cat is dead.

Applying its dual truth functionality pre-decoherence of the Schrodinger's equation would yield:

4 The cat is dead is both true and false.

which can be restated, invoking metalanguage, to:

5 The cat is dead is both true and false is true.

There are possible more coherent methods to restate (5)—for starters, we can use θ to refer to statement (3):

6 θ is both true and false is true.

which we will retain through the course of this paper. Admittedly, (6) is not self-referential and thus not entirely similar to Tarski's position; we shall argue that statements of this kind still benefit from its truth appendages.

One can derive as well through the progression of above statements that (3) is a prediction and doesn't warrant a truth value until further verification. Unless a statement is verified, its truth value is tentatively suspended as well. Such condition takes into consideration wave-function collapse which only occurs in conjunction with time. That which is deemed true, then, is the prevalent outcome resulting after the collapse. Following such reasoning, statement such as "The quanta will be at location *L* given time *T*." cannot be attributed a determinate truth value due to presuppository lapses in computation, unless it is 100% guaranteed that said particle will be found at said location given said time. Say, on the other hand, the sentence "My child has 75% chance of being a twin, hence I will have twins.", which is a compound statement whose antecedent's truth value appears to hinge on the figure stated on the first part. It would be precise to state the second part as "It is highly probable that my child will be a twin", as it is eligible for a truth value.

Given above examples, one can decide whether a statement is absolutely true if they allude to its coordinates, as per possible worlds. In the non-actualized eigenstates, that is, a quantized system but without a determinate value, truth encompass all possible ramifications resulting from an eigenvalue substitution. This means that truth is probabilistic from the perspective of an observer plugging in the eigenvalues, but that *those* probabilities are still subjected under the greater umbrella of whether they are true or not.

NATURE OF THE METALANGUAGE

Not much was said by Tarski on the nature of metalanguage; however, it was stressed at the earlier parts of the discussion that a metalanguage must be richer than the object language (or at least, in terms of its capacity to warrant elimination of paradoxes in the object language). Tarski's notion put forward an immediate appeal to a higher language in order to ascertain self-references and to quiet the Liar, yet the very same process is in danger of invoking *ad nauseum* warrant in the event a paradox occurs.[8] To be sure, reductionists/deflationists might easily dismiss the idea; however, upon treating language as a closed system, Gödel's Incompleteness Theorems answer for the inadequacy and inadvertently, invocation of a metasystem.[9]

Due to lack of details about the possible nature of the metasystem, we now turn to mathematics to aid our reason. A metalanguage must be able to weave through the intricacies of a paradox, indicating that it must have proper instruments (or at least, additional axioms) which will not make it susceptible to such antinomies. Assuming that our model language is the object language, then, the metalanguage then should be equipped with a more definitive notion of truth. As such, it must also be able to justify the axioms corresponding to the model language in order to fully encapsulate and explain the former. As hierarchies rise and possible metalanguages spring forth, we are now woven into an order of "infinities" reminiscent of the Cantorian Set Theory.[10] Metalanguages are greater in

[8] Infinite recurrence of metalanguages subsuming "lower" orders appears to be a very real possibility given this.

[9] Gödel's Incompleteness Theorem, as introduced by the mathematician Kurt Gödel, stipulates two positions: first, that any axiomatically-adherent formal system in any branch of mathematics will not be able to exhaust all true statements within said system; and second, that any formal, arithmetical system cannot prove its own consistency. See Raatikainen, Panu, "Gödel's Incompleteness Theorems", *The Stanford Encyclopedia of Philosophy* (Spring 2021 Edition), Edward N. Zalta (ed.), URL = https://plato.stanford.edu/archives/spr2021/entries/goedel-incompleteness/ accessed October 12, 2021

[10] The Cantorian Set Theory lays down the foundation of the field of set theory in mathematics. It exhibits the importance of a one-to-one correspondence of individual

cardinality ('scope'), yet both systems are infinite in nature (pertaining to endless creation of new terms, which ascends immediately upon introduction to object language). Through this, we eliminate the possibility of the metalanguage being poorer than the object language.

The Importance of Distinguishing Truth from Falsity in Quantum Statements

As with self-referencing statements, a truth value of "false" dissolves the idea corresponding to the initial utterance. To wit,

7 "This statement is false" is false.

merely ensures that above statement is true; and

8 "This statement is true" is false.

is a long hand version of (8.5)

(8.5) *That statement* is false.

In instances reflecting dual truth values, as with quantum mechanical probabilistic statements, the truth value of a probabilistic statement being "false" does not follow from its half-truth. For instance,

9 (4) is false;

elements given two (or more) sets. One of the innovative ramifications thus of said theorem is its position on infinite sets—the cardinality of some sets containing infinity is larger than others, e.g., the set of whole numbers versus the set of integers. This is said to be parallel to our discussion on metalanguages and object languages in terms of their respective sets' cardinality.

is cognitively different from (5). (9) interpolates two interpretations, namely: a) following De Morgan's Theorem,[11] the statement takes the value of not true or not false, in which case can be simplified by virtue of negation into true or false, and b) the entire "true and false" truth value altogether will be taken as one and negated, in which case we have to appeal to another world where falsity is inflated the same way truth is. Should we take option a) for instance, the ensuing condition doesn't convey the initial condition, in the very same manner that true AND false doesn't equate to true OR false. One might be prone to misconstruing the original sentiment of the statement, which is invoking a simultaneous truth and falsity, if they were to heed this interpretation.

On the other hand, (b) analogously calls into mind Tarski's position on truth. By stipulating the existence of a metalanguage in which falsity is more encompassing, we avoid misinterpretations attributed to (a), but with a great cost. Such assessment would then lead us to assume two feasible paths; (i) that the metalanguage in which above falsity coheres to is the same metalanguage which justifies (9), or (ii) that the metalanguage for falsity denotes the notion which would eventually lead to negative existence. Either position is contestable and can be grounds for further researches.

Would a metalanguage appeal as well to possible worlds? We assert that it *can,* following subsequent approaches: (1) we exclude so- called *impossible worlds* and adopt a Tractarian perspective in relation to such; (2) we assume that the elements of our set have a notion of truth which corresponds to ours—meaning, whichever are empirically manifestable in their world would be deemed as truth (adopting the rigid designator model as proposed by Kripke), leaving abstracted worlds whose inclusion lies in a paraconsistent upholding of *a priori* truths; and (3) we assume that at a

[11] One of the basic axioms under the rules of replacement, De Morgan's Theorem states that: ~(A v B) ⇔ (~A ^ ~B) and ~(A ^ B) ⇔ (~A v ~B). We can formalize above example using the simplified denotion (D ^ ~D) where D stands for "The cat is dead is true" (note that we are using truth as an implied predicate here). Negating said statement will yield ~(D^ ~D) ⇔ ~D v ~(~)D, a logical tautology care of the law of excluded middle.

certain extent, a possible world is known to us in as much as we can cover it via systems of modal logic.

TRUTH, MODALITY, AND TARSKI

Orthocomplemented lattices are bound to have least element 0 and most element 1 with an infinity of numbers in between. We can, for ease's sake, take 0 then as "false" and 1 as "true", with the numbers in between ranging from almost, nearly, semi, and least; quantified in terms of range. Statement (4) thus is definable in this schema, lying in the middle of both ends. As per above discussion, we have determined that prior measurement, the statement assumes both "True" and "False" truth value and will remain as is until it becomes observable or decoherence strikes.[12] Appealing to the Tarskian Model of metalinguistic truth, it should be sensible to say that statement (4)'s truth values are bounded as well in the object language system. While not necessarily pertaining to consequences, one should be able to utter (5) in full confidence. Similarly, probabilistic systems such as quantum logic do not transcend to metalanguage, for the metalanguage validates said system. Invoking the Incompleteness theorem, logic systems of any kinds can only be validated by a system greater than such, which would in turn necessitate the creation of a higher system to validate previous system.

Language and Form

Now a certain problem arises as we try to bridge a linguistic and a formal system. Gupta conceives two concepts of truth following his efforts of ascertaining truth in modal logic. He proposed the notion of T_1 which

[12] Note that the conjunction is used in this example as we assume that the usage of an alternative (i.e., disjunction) presupposes in a way that the wave function has already collapsed. It is, as I believe, more apt to use a conjunction in order to capture the comprehensiveness of the event; there is no need to deal with mutually exclusive and inclusive events in conjunction as opposed to disjunction.

denotes truth of a statement if and only if it corresponds to its meaning in world w, and T_2 which "argues for the truth of a statement if and only if it corresponds to its meaning in the actual world."[13] We thus are able to differentiate axiomatic contradictions as opposed to contingent contradictions, such as his example:

A All contradictions are necessarily false.

B 'Snow is white, or it is not white' is necessarily true (in English).[14]

We may view this as two systems each with potential metalanguages. As the scope of possible worlds are vast in nature, one cannot reject contradicting true statements fully. Are there possible worlds where empirical evidences do not correspond to the truth (or, in other words, "The snow is white" or its counterpart clearly manifesting in such world cannot be said as true)? If yes, then we should subscribe to a different notion. If no, then truth is something which prevail over worlds. The first option is already dismissed as per qualifying included worlds in our set, which leaves us with the second. If truth, thus, is something which prevail over worlds, then said worlds must have a methodology closely resembling ours in determining which statements are true. We thus call these true statements in other possible worlds *shtatements* for proper referencing. Such shtatements then are deemed true if (1) they are true *a priori;* (2) they correspond to a state of affair which occur within said world. Quantum possible worlds are subsumed under S4 system as per Nino Cocchiarella (2010).[15] Implying that there are certain necessary conditions which

[13] A. Gupta, "Modal Logic and Truth," *Journal of Philosophical Logic,* 7:1 (1978), 453.
[14] *Ibid.*
[15] N. Cocchiarella, "Actualism versus Possibilism in Formal Ontology" in Johanna Seibt and Roberto Poli (eds.), *Theory and Applications of Ontology: Philosophical Perspectives* (New York: Springer, 2010).

prevail among all possible worlds, as wave-function collapse is partly causal in nature, possible worlds thus are not entirely stochastic. However, the question remains as to what exactly an S4 system of possible worlds entail.

Further Considerations

While this might be implied in the previous part of the paper, a statement being true in a possible world means that, in a quantum perspective, an observer (consciousness or minds) is present to determine that such is the case. So much has yet to be said regarding this phenomenon, and the debate among objectivity, subjectivity, and intersubjectivity of truth is a topic the paper wishes to steer clear off of. It cannot be disputed however that consciousness, an aspect outside the quantum system, grants the system stability and causes decoherence to collapse. The subscription on a system outside the closed system to determine its validity closely resembles Gödel's Incompleteness Theorem and Tarski's Metalanguage, sans assumption of the universe's being a closed one. While one might be pushing the analogy too far, it is, at the very least, a curious thought to entertain—that the metalanguage of reality is in fact, consciousness.

Another equally important discussion that arises is the definition of a true statement, which remains untackled in this paper. Tarski's metalanguage addresses the definition of truth in language; however, its actual extension on the grounds of what makes a statement true in the metalanguage is left wanting.[16] Due to possible peculiarities governing the notion and composition of a true sentence, we might have to entertain the idea that the upward recursion to a cardinally-greater metalanguage might not always be the way to go since truth-conditions might not necessarily hinge on individual elements or words. This would thus imply that the Cantorian referencing method might not hold always, paving way for a "lesser" metalanguage.

[16] Jeroen Smid, "Tarski's One and Only Concept of Truth," *Synthese*, 191:14 (2014), 3393-3406.

CONCLUSION

Probabilistic and multi-functional statements yielded through quantum mechanics can be subsumed and essentially, cognized if the assessment "is true" is appended. This distinctively differs from appendage of "is false", regardless of the statement's partial assumption of said truth value. This is further justified by appealing to Tarski's method of subscribing to a metalanguage to remove antinomies and paradoxes in language, such as the Liar paradox and the self-referencing statement. Truth is, as opposed to deflationist position, not redundant nor vestigial when appended to like statements, as the percentages of likelihood are partial appeal to the self-referencing false statement. In general, since our understanding of reality grows exponentially, we should up ante our treatment of truth to keep up with the lightspeed advancements of breakthroughs in reality. Should truth be confined in its classical sense, it will fail to uphold above notion. Admittedly perhaps that our common sense cannot fathom said discoveries; perhaps the limitation thus of truth is not to be blamed to its shortcomings but our bounded comprehension of what truth is. Language, perhaps, should also be radically modified in order to cater to new discoveries about reality.

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